appendix b

APPENDIX B

NONPARA Computer Program Description

This appendix briefly summarizes the functions evaluated in the NONPARA program, and gives the methods used to determine their statistical significance.

GENERAL

Any statistical test of significance will generally be made using the following steps:

a) State the null hypothesis, H_o. For instance, in split sample tests, the null hypothesis may be that there is no difference between the sample means.

b) Choose a significance level, α.

c) Choose an appropriate statistical test. In this program, all tests are non-parametric.

d) Compute the test statistic.

e) The sampling distribution of the test statistic is known and has been tabulated, and the chosen significance level then defines the region of rejection.

f) If the computed test statistic lies in the region of rejection, then the null hypothesis is rejected.

Considering now the five tests in this program.

1) The Spearman rank order serial correlation coefficient for independence.

If the series Q_i with i ranging from 1 to N is put in chronological order, ranks assigned, and denoting the series

$Q_1, Q_2 \ldots\ldots\ldots Q_{N-1}$ by x_i, the rank of Q_i, and

$Q_2, Q_3 \ldots\ldots\ldots Q_N$ by y_i, the rank of Q

then the Spearman rank order serial correlation coefficient is

$$S_1 = 1/2 \ (\sum x_i^2 + \sum y_i^2 - \sum d_i^2) \ (\sum x_i^2 \ \sum y_i^2)^{-1/2} \qquad 1.1$$

where $\sum x_i^2 = (m^3 - m)/12 - \sum T_x$

$\sum y_i^2 = (m^3 - m)/12 - \sum T_y$

d_i is the difference in rank between x_i and y_i

$m = N-1$

and the summations are over the m pairs of x_i, y_i

Ignoring for the moment the terms in T and putting them at zero, equation 1.1 becomes

$$S_1 = 1 - (6\sum d_1^2)/(m^3 - m) \qquad 1.2$$

the more familiar form of the Spearman rank correlation coefficient.

The terms in T adjust for tied ranks and are computed as follows. If for instance three observations in the x series were tied for ranks 17, 18 and 19, then each observation is given the rank 18; if two were tied for ranks 24 and 25, then each is ranked 24.5.

For each tied set, T is computed from

$$T_x = (t^3 - t)/12$$

where t is the number of observations tied at a given rank.

$\sum T_x$ and $\sum T_y$ are defined by extension of the foregoing.

For N less than 10, special tables are available for defining the region of rejection for a computed S_1 at given significance level α. When N is 10 or greater, then the function:

$$t = s_1[(m - 2)/(1 - s_1^2)]1/2 \qquad 1.3$$

is distributed like Student's t with m-2 degrees of freedom. A one-tail test must be used.

2) The Spearman rank order correlation coefficient test for trend.

If the series Q_i with i ranging from 1 to N is put in chronological order, ranks assigned and denoting the series

$Q_1, Q_2 \ldots\ldots\ldots\ldots Q_N$ by y_i, the rank of Q_i and

$1, 2, \ldots\ldots\ldots\ldots N$ by x_i, the sequential order of Q_i

then the Spearman rank order correlation coefficient r_s is calculated as in equation 1.1, except that m = N, $T_x = 0$, and the summations are taken over the N pairs of x_i, y_i.

For N less than 10, special tables define the region of rejection for a computed value of r_s at a given significance level α.

For N = 10 or greater, then the function

$$t = r_s[(N - 2)/(1 - r_s^2)]1/2 \qquad 2.1$$

is distributed like Student's t with N-2 degrees of freedom. The null hypothesis is that there is no trend, either upward or downward with time, and so a two-tail test is used.

3) Mann-Whitney split sample test for homogeneity.

As described in the Appendix A, page A(2), the sample is split into two sub-samples, and ranks assigned. Then the Mann-Whitney U statistic is defined by the smaller of

$$U_1 = n_1 n_2 + n_1(n_1 + 1)/2 - R_1 \qquad 3.1$$

$$\text{or } U_2 = n_1 n_2 - U_1 \qquad 3.2$$

where n_1 is the size of the smaller sub-sample

n_2 is the size of the larger sub-sample

R_1 is the sum of the ranks in sub-sample n_1

For both n_1 and n_2 less than 21, the critical values of U have been tabulated which define the region of rejection. For n_1 greater than 4 and n_2 greater than 20, the sampling distribution of U rapidly tends to normality with

$$z = \frac{U - n_1 n_2/2}{\left\{\left[\frac{n_1 n_2}{N(N-1)}\right]\left[\frac{N^3 - N}{12}\right] - \sum T\right\}^{1/2}} \qquad 3.3$$

$T = (t^3 - t)/12$, where t is the number of observations tied at a given rank. The summation of T is over all groups of tied observations in both sub-samples.

z is an N (0,1) variate and in the applications of the Mann-Whitney test used in this program, the region of rejection is

z less than - 1.645 for $\alpha = 0.05$
z less than - 2.326 for $\alpha = 0.01$

4) Wald-Wolfowitz split sample test for homogeneity.

Having determined the number of runs, R_{ww}, as explained on page (3), the method by which its significance is determined depends on the sub-samples sizes, n_1 and n_2. When both n_1 and n_2 are less than 21, the critical values of R_{ww} d which define the region of rejection have been tabulated. For n_1 greater than 4 and n_1 greater than 20, the sampling distribution of R_{ww} tends to normality with

$$z = \frac{[R_{ww}-[(2n_1n_2)/(n_1+n_2)+1] -0.5]}{[2n_1n_1(2n_1n_2-n_1-n_1)/[(n_1+n_1)2(n_1+n_1-1)]]^{1/2}} \quad 4.1$$

z is an N (0,1) variate, and in the applications of the Wald-Wolfowitz test used herein, the region of rejection is

z greater than 1.645 for $\alpha = 0.05$
z greater than 2.326 for $\alpha = 0.01$

Theoretically ties cannot occur in the Wald-Wolfowitz test since in its derivation the samples are assumed to be drawn from continuous distributions. In hydrologic practice, published values of flows have been rounded to comply with some rule for significant figures and ties are very common. If for any tied group, all members are in the same sub-sample there is no problem, but if members of one sub-sample are tied with members of the other sub-sample, then there is no unique ordered series and hence no unique value of R_{WW}. For instance, if a quartet of ties had two members in each sub-sample and a duo of ties had one member in each sub-sample, then there are 12 possible ordered series and the test becomes meaningless. In this program, if ties are split between sub-samples, the Wald-Wolfowitz statistic is not computed.

5) Runs above and below the median for general randomness.

Page 3 explains how the number of runs, RUNAB is determined, and for n_1 A's and n_2 B's with n_1 and n_2 both less than 21, the region of rejection is defined by tables. For n_1 and n_2 both greater than 20, the sampling distribution of RUNAB tends to normality with

$$z = \frac{\left| \text{RUNAB} - [(2n_1n_2)/(n_1+n_2)+1] \right|}{\left\{2n_1n_2(2n_1n_2-n_1-n_2)/[(n_1+n_2)2(n_1+n_2-1)]\right\}^{1/2}} \qquad 5.1$$

z is an N (0,1) variate and as used in this program, the region of rejection is

z greater than 1.96 for $\alpha = 0.05$

z greater than 2.326 for $\alpha = 0.01$